

# Module 1

<b>Anchor</b>	<b>Lessons</b>
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<b>A1.1.2 – Linear Equations</b>	<b>3.1 – 3.5 – Write, Solve, and Apply Linear Equations</b> <hr/> <b>2.5, 2.6 – Use and/or Identify an Algebraic Property</b> <hr/> <b>5.5 – Interpret Solutions to Problems</b> <hr/> <b>7.1 – 7.3 – Write and/or Solve a System of Linear Equations</b> <hr/> <b>7.4 – Interpret Solutions to Problems</b>
<b>A1.1.3 – Linear Inequalities</b>	<b>6.1, 6.2, 6.4 – 6.6 – Write, Solve, and Graph Linear Inequalities</b> <hr/> <b>Solve Systems of Linear Inequalities</b>

**Practice with Examples**

For use with pages 65–70

**GOAL** Graph, compare, and order real numbers.**VOCABULARY**

Real numbers are the positive numbers, the negative numbers, and zero.

The real number line is a line whose points correspond to the real numbers.

Negative numbers are numbers less than zero.

Positive numbers are numbers greater than zero.

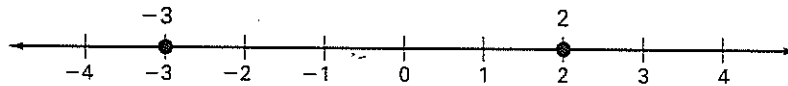
Integers are the numbers ..., -3, -2, -1, 0, 1, 2, 3 ....

Whole numbers are the positive integers and zero.

The graph of a number is the point that corresponds to the number.

**EXAMPLE 1** *Graphing and Comparing Integers*

Graph -3 and 2 on a number line. Then write two inequalities that compare the two numbers.

**SOLUTION**

On the graph, -3 is to the left of 2, so -3 is less than 2. You can write this using the symbols:

$$-3 < 2$$

On the graph, 2 is to the right of -3, so 2 is greater than -3. You can write this using the symbols:

$$2 > -3$$

**Exercises for Example 1**

Graph the numbers on a number line. Then write two inequalities that compare the numbers.

1. -4 and -7

2. -8 and 9

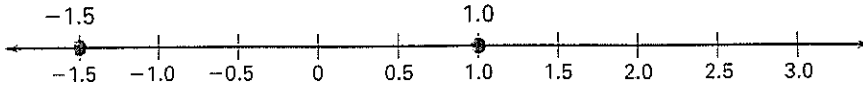
3. 5 and -6

**Practice with Examples**

For use with pages 65–70

**EXAMPLE 2** *Graphing Real Numbers*

Graph  $-1.5$  and  $1.0$  on a number line. Then write two inequalities that compare the two numbers.

**SOLUTION**

On the graph,  $-1.5$  is to the left of  $1.0$ , so  $-1.5$  is less than  $1.0$ .

$$-1.5 < 1.0$$

On the graph,  $1.0$  is to the right of  $-1.5$ , so  $1.0$  is greater than  $-1.5$ .

$$1.0 > -1.5$$

**Exercises for Example 2**

Graph the numbers on a number line. Then write two inequalities that compare the two numbers.

4.  $-2.3$  and  $-2.8$

5.  $-5.2$  and  $-4.8$

6.  $-0.6$  and  $0.3$

## Practice with Examples

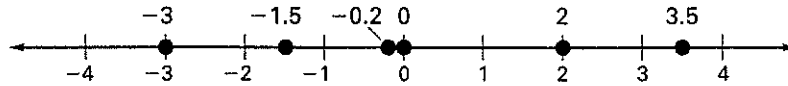
For use with pages 65–70

### EXAMPLE 3 Ordering Real Numbers

Write the following numbers in increasing order: 3.5,  $-3$ , 0,  $-1.5$ ,  $-0.2$ , 2.

#### SOLUTION

First graph the numbers on a number line.



From the graph, you can see that the order is  $-3$ ,  $-1.5$ ,  $-0.2$ ,  $0$ ,  $2$ ,  $3.5$ .

#### Exercises for Example 3

Write the numbers in increasing order.

7. 2,  $-3$ ,  $-2.5$ ,  $4.5$ ,  $-1.5$

8.  $-\frac{3}{4}$ ,  $\frac{1}{4}$ ,  $-2$ ,  $-\frac{5}{4}$ , 1

**Practice with Examples**

For use with pages 499–504

**GOAL** Evaluate and approximate square roots**VOCABULARY**If  $b^2 = a$ , then  $b$  is a **square root** of  $a$ .A square root  $b$  can be a **positive square root** (or a principal square root) or a **negative square root**.A **radicand** is the number or expression inside a radical symbol  $\sqrt{\quad}$ .**Perfect squares** are numbers whose square roots are integers.A number that is not the quotient of integers is an **irrational number**.A **radical expression** is an expression written with a radical symbol.**EXAMPLE 1** *Finding Square Roots of Numbers*

Evaluate the expression.

a.  $\sqrt{81}$

b.  $-\sqrt{49}$

c.  $\pm\sqrt{16}$

d.  $\sqrt{0}$

**SOLUTION**

a.  $\sqrt{81} = 9$  Positive square root

b.  $-\sqrt{49} = -7$  Negative square root

c.  $\pm\sqrt{16} = \pm 4$  Two square roots

d.  $\sqrt{0} = 0$  Square root of zero is zero.

**Practice with Examples**

For use with pages 499–504

**Exercises for Example 1**

Evaluate the expression.

1.  $\sqrt{9}$

2.  $\sqrt{36}$

3.  $-\sqrt{25}$

4.  $\pm\sqrt{100}$

**EXAMPLE 2** *Evaluating a Radical Expression*Evaluate  $\sqrt{b^2 - 4ac}$  when  $a = -2$ ,  $b = -5$ , and  $c = 2$ .**SOLUTION**

$$\begin{aligned} \sqrt{b^2 - 4ac} &= \sqrt{(-5)^2 - 4(-2)(2)} && \text{Substitute values.} \\ &= \sqrt{25 + 16} && \text{Simplify.} \\ &= \sqrt{41} && \text{Simplify.} \\ &\approx 6.40 && \text{Round to the nearest hundredth.} \end{aligned}$$

## ***Practice with Examples***

For use with pages 499–504

### ***Exercises for Example 2***

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Evaluate  $\sqrt{b^2 - 4ac}$  for the given values.

5.  $a = -3, b = 6, c = -3$

6.  $a = 1, b = 5, c = 4$

**Practice with Examples**

For use with pages 15–21

**GOAL**

Use the established order of operations.

**VOCABULARY**

An established order of operations is used to evaluate an expression involving more than one operation.

**EXAMPLE 1****Evaluate Expressions Without Grouping Symbols**

- a. Evaluate  $5x^2 - 6$  when  $x = 3$ . Use the order of operations.  
 b. Evaluate  $7 + 15 \div 3 - 4$ . Use the order of operations.

**SOLUTION**

$$\begin{aligned} \text{a. } 5x^2 - 6 &= 5 \cdot 3^2 - 6 \\ &= 5 \cdot 9 - 6 \\ &= 45 - 6 \\ &= 39 \end{aligned}$$

Substitute 3 for  $x$ .

Evaluate power.

Evaluate product.

Evaluate difference.

$$\begin{aligned} \text{b. } 7 + 15 \div 3 - 4 &= 7 + (15 \div 3) - 4 \\ &= 7 + 5 - 4 \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

Divide first.

Evaluate quotient.

Work from left to right.

Evaluate difference.

**Exercises for Example 1**

Evaluate the expression.

1.  $4 \cdot 3 + 8 \div 2$

2.  $24 \div 6 \cdot 2$

3.  $21 - 5 \cdot 2$





## Practice with Examples

For use with pages 15–21

### EXAMPLE 2 Evaluate Expressions With Grouping Symbols.

Evaluate  $24 \div (6 \cdot 2)$ . Use the order of operations.

#### SOLUTION

$$\begin{aligned} 24 \div (6 \cdot 2) &= 24 \div 12 \\ &= 2 \end{aligned}$$

Simplify  $6 \cdot 2$ .

Evaluate the quotient.

#### Exercises for Example 2

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Evaluate the expression.

4.  $(6 - 2)^2 - 1$

5.  $30 \div (1 + 4) + 2$

6.  $(8 + 4) \div (1 + 2) + 1$

7.  $6 - (2^2 - 1)$

8.  $(30 \div 1) + (4 + 2)$

9.  $8 + 4 \div (1 + 2 + 1)$

## Practice with Examples

For use with pages 15–21

### EXAMPLE 3 Calculate Family Admission Prices

Use the table below which shows admission prices for a theme park. Suppose a family of 2 adults and 3 children go to the park. The children's ages are 6 years, 8 years, and 13 years.

- Write an expression that represents the admission price for the family.
- Use a calculator to evaluate the expression.

<i>Theme Park Admission Prices</i>	
<i>Age</i>	<i>Admission Price</i>
Adults	\$34.00
Children (3–9 years)	\$21.00
Children (2 years and under)	free

#### SOLUTION

- The admission price for the child who is 13 years old is \$34, the adult price. The family must buy 3 adult tickets and 2 children's tickets. An expression that represents the admission price for the family is  $3(34) + 2(21)$ .
- If your calculator uses the established order of operations, the following keystroke sequence gives the result 144.

3  $\times$  34  $+$  2  $\times$  21 ENTER

The admission price for the family is \$144.

#### Exercise for Example 3

- Rework Example 3 for a family of 2 adults and 4 children. The children's ages are 2 years, 4 years, 10 years, and 12 years.

## Practice with Examples

For use with pages 71–76

**GOAL** Find the opposite and absolute value of a number.

### VOCABULARY

**Opposites** are two numbers that are the same distance from zero on a number line but on opposite sides.

The **absolute value** of a real number is its distance from zero on a number line.

A **counterexample** is a single example used to prove that a statement is false.

### **EXAMPLE 1** *Finding the Opposite and the Absolute Value of a Number*

- Find the opposite of each of the numbers 3.6 and  $-7$ .
- Find the absolute value of each of the numbers 3.6 and  $-7$ .

### SOLUTION

- The opposite of 3.6 is  $-3.6$  because each is 3.6 units from zero.  
The opposite of  $-7$  is 7 because each is 7 units from zero.
- The absolute value of 3.6 is 3.6. The absolute value of  $-7$  is 7 because the absolute value of a number represents distance, which is never negative.

### *Exercises for Example 1*

Find the opposite of the number. Then find the absolute value of the number.

1.  $-1.7$

2. 4.2

3.  $-5$

## Practice with Examples

For use with pages 71–76

### EXAMPLE 2 Solve an Absolute Value Equation

Use mental math to solve the equation. If there is no solution, write *no solution*.

a.  $|x| = 3$

b.  $|x| = -0.5$

c.  $|x| = \frac{1}{3}$

#### SOLUTIONS

a.  $-3, 3$

b. no solution

c.  $-\frac{1}{3}, \frac{1}{3}$

#### Exercises for Example 2

Use mental math to solve the equation. If there is no solution, write *no solution*.

4.  $|x| = 1.8$

5.  $|x| = -14$

6.  $|x| = 11.3$

### EXAMPLE 3 Finding Velocity and Speed

An elevator descends at a rate of 900 feet per minute. Find the velocity and the speed of the elevator.

#### SOLUTION

Velocity =  $-900$  ft per min

Motion is downward.

Speed =  $|-900| = 900$  ft per min

Speed is positive.

#### Exercises for Example 3

Find the speed and the velocity of the object.

7. A duck hawk descends at 150 miles per hour when striking its prey.

8. A helicopter descends at a rate of 8 feet per second.

**Practice with Examples**

For use with pages 71–76

**EXAMPLE 4** *Use a Counterexample*

Determine whether the statement is *true* or *false*. If it is false, give a counterexample.

- a. The quotient of a number and its opposite is always equal to 1.
- b. The expression  $\frac{|a|}{a}$  is always equal to  $-1$ .

**SOLUTION**

- a. False. Counterexample:  $\frac{5}{-5} = -1$
- b. False. Counterexample: If  $a = 5$ , then  $\frac{|5|}{5} = \frac{5}{5} = 1$ .

**Exercises for Example 4**

Determine whether the statement is true or false. If it is false, give a counterexample.

9. The sum of a number and its opposite is always 0.
10. The product of a number and its opposite is always negative.
11. The absolute value of a negative fraction is always greater than 1.

## Practice with Examples

For use with pages 441–448

**GOAL** Use multiplication properties of exponents.

### VOCABULARY

Let  $a$  and  $b$  be real numbers and let  $m$  and  $n$  be positive integers.

#### Product of Powers Property

To multiply powers having the same base, add the exponents.

$$a^m \cdot a^n = a^{m+n} \quad \text{Example: } 3^2 \cdot 3^7 = 3^{2+7} = 3^9$$

#### Power of a Power Property

To find a power of a power, multiply the exponents.

$$(a^m)^n = a^{m \cdot n} \quad \text{Example: } (5^2)^4 = 5^{2 \cdot 4} = 5^8$$

#### Power of a Product Property

To find a power of a product, find the power of each factor and multiply.

$$(a \cdot b)^m = a^m \cdot b^m \quad \text{Example: } (2 \cdot 3)^6 = 2^6 \cdot 3^6$$

**EXAMPLE 1** Using the Product of Powers Property

a.  $4^3 \cdot 4^5$

b.  $(-x)(-x)^2$

### SOLUTION

To multiply powers having the same base, add the exponents.

$$\begin{aligned} \text{a. } 4^3 \cdot 4^5 &= 4^{3+5} \\ &= 4^8 \end{aligned}$$

$$\begin{aligned} \text{b. } (-x)(-x)^2 &= (-x)^1(-x)^2 \\ &= (-x)^{1+2} \\ &= (-x)^3 \end{aligned}$$

### Exercises for Example 1

Use the product of powers property to simplify the expression.

1.  $m \cdot m$

2.  $6^2 \cdot 6^3$

3.  $y^4 \cdot y^3$

4.  $3 \cdot 3^5$

**Practice with Examples**

For use with pages 441–448

**EXAMPLE 2** *Using the Power of a Power Property*

a.  $(z^4)^5$

b.  $(2^3)^2$

**SOLUTION**

To find a power of a power, multiply the exponents.

$$\begin{aligned} \text{a. } (z^4)^5 &= z^{4 \cdot 5} \\ &= z^{20} \end{aligned}$$

$$\begin{aligned} \text{b. } (2^3)^2 &= 2^{3 \cdot 2} \\ &= 2^6 \end{aligned}$$

**Exercises for Example 2**

Use the power of a power property to simplify the expression.

5.  $(w^7)^3$

6.  $(7^3)^5$

7.  $(t^2)^6$

8.  $[(-2)^3]^2$

**EXAMPLE 3** *Using the Power of a Product Property*Simplify  $(-4mn)^2$ .**SOLUTION**

To find a power of a product, find the power of each factor and multiply.

$$\begin{aligned} (-4mn)^2 &= (-4 \cdot m \cdot n)^2 && \text{Identify factors.} \\ &= (-4)^2 \cdot m^2 \cdot n^2 && \text{Raise each factor to a power.} \\ &= 16m^2n^2 && \text{Simplify.} \end{aligned}$$



## Practice with Examples

For use with pages 441–448

### Exercises for Example 3

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Use the power of a product property to simplify the expression.

9.  $(5x)^3$

10.  $(10s)^2$

11.  $(-x)^4$

12.  $(-3y)^3$

### EXAMPLE 4 Using Powers to Model Real-Life Problems

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You are planting two square vegetable gardens. The side of the larger garden is twice as long as the side of the smaller garden. Find the ratio of the area of the larger garden to the area of the smaller garden.

#### SOLUTION

$$\text{Ratio} = \frac{(2x)^2}{x^2} = \frac{2^2 \cdot x^2}{x^2} = \frac{4x^2}{x^2} = \frac{4}{1}$$

### Exercise for Example 4

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13. Rework Example 4 if the side of the larger garden is three times as long as the side of the smaller garden.



**Practice with Examples**

For use with pages 449–454

**GOAL**

Evaluate powers that have zero or negative exponents.

**VOCABULARY**Let  $a$  be a nonzero number and let  $n$  be an integer.

- A nonzero number to the zero power is 1:  $a^0 = 1$ ,  $a \neq 0$ .
- $a^{-n}$  is the reciprocal of  $a^n$ :  $a^{-n} = \frac{1}{a^n}$ ,  $a \neq 0$ .

**EXAMPLE 1****Powers with Zero and Negative Exponents**

Evaluate the exponential expression.

a.  $(-8)^0$

b.  $4^{-2}$

**SOLUTION**

a.  $(-8)^0 = 1$       A nonzero number to the zero power is 1.

b.  $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$        $4^{-2}$  is the reciprocal of  $4^2$ .

**Exercises for Example 1**

Evaluate the exponential expression.

1.  $73^0$

2.  $\left(\frac{1}{2}\right)^{-1}$

3.  $13^{-2}$

**Practice with Examples**

For use with pages 449–454

**EXAMPLE 2** *Simplifying Exponential Expressions*

Rewrite the expression with positive exponents.

a.  $5y^{-1}z^{-2}$

b.  $(2x)^{-3}$

**SOLUTION**

a.  $5y^{-1}z^{-2} = 5 \cdot \frac{1}{y} \cdot \frac{1}{z^2} = \frac{5}{yz^2}$

b.  $(2x)^{-3} = 2^{-3} \cdot x^{-3}$  Use power of a product property.

$$= \frac{1}{2^3} \cdot \frac{1}{x^3}$$
 Write reciprocals of  $2^3$  and  $x^3$ .

$$= \frac{1}{8x^3}$$
 Multiply fractions.

**Exercises for Example 2**

Rewrite the expression with positive exponents.

4.  $(13y)^{-1}$

5.  $\frac{1}{(2x)^{-4}}$

6.  $(2c)^{-4}d$

**Practice with Examples**

For use with pages 449–454

**EXAMPLE 3** *Evaluating Exponential Expressions*

Evaluate the expression.

a.  $(3^{-2})^{-3}$

b.  $2^{-2} \cdot 2^{-1}$

**SOLUTION**

a.  $(3^{-2})^{-3} = 3^{-2 \cdot (-3)}$

$= 3^6$

$= 729$

Use power of a power property.

Multiply exponents.

Evaluate.

b.  $2^{-2} \cdot 2^{-1} = 2^{-2+(-1)}$

$= 2^{-3}$

$= \frac{1}{2^3}$

Use product of powers property.

Add exponents.

 $2^{-3}$  is the reciprocal of  $2^3$ .

$= \frac{1}{8}$

Evaluate power.

**Exercises for Example 3**

Evaluate the expression.

7.  $8^{-1} \cdot 8^1$

8.  $4^6 \cdot 4^{-4}$

9.  $(5^{-2})^2$

**Practice with Examples**

For use with pages 462–468

**GOAL** Use division properties of exponents.**VOCABULARY**Let  $a$  and  $b$  be real numbers and let  $m$  and  $n$  be integers.**Quotient of Powers Property**

To divide powers having the same base, subtract exponents.

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0 \quad \text{Example: } \frac{3^7}{3^5} = 3^{7-5} = 3^2$$

**Power of a Quotient Property**

To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0 \quad \text{Example: } \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$$

**EXAMPLE 1** *Using the Quotient of Powers Property*

Use the quotient of powers property to simplify the expression.

a.  $\frac{8^2 \cdot 8^4}{8^3}$

b.  $z^7 \cdot \frac{1}{z^8}$

**SOLUTION**

To divide powers having the same base, subtract exponents.

$$\begin{aligned} \text{a. } \frac{8^2 \cdot 8^4}{8^3} &= \frac{8^6}{8^3} \\ &= 8^{6-3} \\ &= 8^3 \end{aligned}$$

$$\begin{aligned} \text{b. } z^7 \cdot \frac{1}{z^8} &= \frac{z^7}{z^8} \\ &= z^{7-8} \\ &= z^{-1} \\ &= \frac{1}{z} \end{aligned}$$

**Exercises for Example 1**

Use the quotient of powers property to simplify the expression.

1.  $\frac{10^4}{10}$

2.  $\frac{3^2}{3^3}$

3.  $\frac{1}{y^2} \cdot y^8$

**Practice with Examples**

For use with pages 462–468

**EXAMPLE 2** *Simplifying an Expression*Simplify the expression  $\left(\frac{7a}{b^2}\right)^3$ .**SOLUTION**

$$\begin{aligned} \left(\frac{7a}{b^2}\right)^3 &= \frac{(7a)^3}{(b^2)^3} && \text{Power of a quotient} \\ &= \frac{7^3 \cdot a^3}{b^6} && \text{Power of a product and power of a power} \\ &= \frac{343a^3}{b^6} && \text{Simplify.} \end{aligned}$$

**Exercises for Example 2**

Simplify the expression.

4.  $\left(\frac{2}{x^3}\right)^4$

5.  $\frac{z \cdot z^5}{z^2}$

6.  $\left(\frac{5y^2}{w}\right)^2$

**Practice with Examples**

For use with pages 567–573

**GOAL**

Add and subtract polynomials.

**VOCABULARY**

A **monomial in one variable** is a number, a variable, or a product of numbers and variables.

A **polynomial** is a monomial or a sum of monomials.

A polynomial is written in **standard form** when the terms are placed in descending order, from largest exponent to smallest exponent.

The **degree** of each term of a polynomial is the exponent of the variable.

The **degree of a polynomial in one variable** is the largest exponent of that variable.

A **binomial** is a polynomial of two terms.

A **trinomial** is a polynomial of three terms.

**EXAMPLE 1****Identifying Polynomials**

Identify the polynomial by degree and by the number of terms.

- $-2$
- $2x^2 - 5$
- $x^3 + x - 8$
- $\frac{1}{2}x$

**SOLUTION**

- constant; monomial
- quadratic; binomial
- cubic; trinomial
- linear; monomial

**Exercises for Example 1**

Identify the polynomial by degree and by the number of terms.

- $5x^3$
- $-x^2 + 4x + 6$
- $-7 + 3x$
- $8x^2$

**Practice with Examples**

For use with pages 567–573

**EXAMPLE 2****Adding Polynomials**

Find the sum and write the answer in standard form.

a.  $(6x - x^2 + 3) + (4x^2 - x - 2)$       b.  $(x^2 - x - 4) + (2x + 3x^2 + 1)$

**SOLUTION**

a. Vertical format: Write each expression in standard form. Line up like terms.

$$-x^2 + 6x + 3$$

$$\underline{4x^2 - x - 2}$$

$$3x^2 + 5x + 1$$

b. Horizontal format: Group like terms.

$$\begin{aligned}(x^2 - x - 4) + (2x + 3x^2 + 1) &= (x^2 + 3x^2) + (-x + 2x) + (-4 + 1) \\ &= 4x^2 + x - 3\end{aligned}$$

**Exercises for Example 2**

Find the sum. Write the answer in standard form.

5.  $(7 + 2x - 4x^2) + (-3x + x^2 - 5)$

6.  $(8x - 9 + 2x^2) + (1 + x - 6x^2)$

## Practice with Examples

For use with pages 567–573

### EXAMPLE 3 Subtracting Polynomials

Find the difference and write the answer in standard form.

a.  $(5x^2 - 4x + 1) - (8 - x^2)$                       b.  $(-x + 2x^2) - (3x^2 + 7x - 2)$

#### SOLUTION

a. Vertical format: To subtract, you add the opposite.

$$\begin{array}{r} (5x^2 - 4x + 1) \\ - (8 - x^2) \\ \hline \end{array} \quad \text{Add the opposite.} \quad \begin{array}{r} 5x^2 - 4x + 1 \\ + x^2 \quad - 8 \\ \hline 6x^2 - 4x - 7 \end{array}$$

b. Horizontal format: Group like terms and simplify.

$$\begin{aligned} (-x + 2x^2) - (3x^2 + 7x - 2) &= -x + 2x^2 - 3x^2 - 7x + 2 \\ &= (2x^2 - 3x^2) + (-x - 7x) + 2 \\ &= -x^2 - 8x + 2 \end{aligned}$$

#### Exercises for Example 3

Find the difference. Write the answer in standard form.

7.  $(x + 7x^2) - (1 + 3x - x^2)$

8.  $(2x + 3 - 5x^2) - (2x^2 - x + 6)$



**Practice with Examples**

For use with pages 574–580

**GOAL**

Multiply polynomials.

**VOCABULARY**

To multiply two binomials, use a pattern called the **FOIL** pattern.  
Multiply the **F**irst, **O**uter, **I**nner, and **L**ast terms.

**EXAMPLE 1****Multiplying Binomials Using the FOIL Pattern**Find the product  $(4x + 3)(x + 2)$ .**SOLUTION**

$$\begin{array}{cccc}
 & \text{F} & \text{O} & \text{I} & \text{L} \\
 & \downarrow & \downarrow & \downarrow & \downarrow \\
 (4x + 3)(x + 2) & = & 4x^2 & + & 8x & + & 3x & + & 6 \\
 & & = & 4x^2 & + & 11x & + & 6 & \quad \text{Combine like terms.}
 \end{array}$$

**Exercises for Example 1**

Use the FOIL pattern to find the product.

1.  $(2x + 3)(x + 1)$

2.  $(y - 2)(y - 3)$

3.  $(3a + 2)(2a - 1)$

**EXAMPLE 2****Multiplying Polynomials Vertically**Find the product  $(x + 3)(4 - 2x^2 + x)$ .**SOLUTION**

To multiply two polynomials that have three or more terms, you must multiply each term of one polynomial by each term of the other polynomial. Align like terms vertically.

$$\begin{array}{r}
 -2x^2 + x + 4 \\
 \times \quad \quad \quad x + 3 \\
 \hline
 -6x^2 + 3x + 12 \\
 -2x^3 + \quad x^2 + 4x \\
 \hline
 -2x^3 - 5x^2 + 7x + 12
 \end{array}$$

Standard form

Standard form

$3(-2x^2 + x + 4)$

$x(-2x^2 + x + 4)$

Combine like terms.

## Practice with Examples

For use with pages 574–580

### Exercises for Example 2

Multiply the polynomials vertically.

4.  $(a + 4)(a^2 + 3 - 2a)$

5.  $(2y + 1)(y^2 - 5 + y)$

### EXAMPLE 3 *Multiplying Polynomials Horizontally*

Find the product  $(x + 4)(-2x^2 + 3x - 1)$ .

#### SOLUTION

Multiply  $-2x^2 + 3x - 1$  by each term of  $x + 4$ .

$$\begin{aligned} (x + 4)(-2x^2 + 3x - 1) &= x(-2x^2 + 3x - 1) + 4(-2x^2 + 3x - 1) \\ &= -2x^3 + 3x^2 - x - 8x^2 + 12x - 4 \\ &= -2x^3 - 5x^2 + 11x - 4 \end{aligned}$$

### Exercises for Example 3

Multiply the polynomials horizontally.

6.  $(a + 4)(a^2 + 3 - 2a)$

7.  $(2y + 1)(y^2 - 5 + y)$

## Practice with Examples

For use with pages 574–580

### EXAMPLE 4 *Multiplying Binomials to Find an Area*

The dimensions of a rectangular garden can be represented by a width of  $(x + 6)$  feet and a length of  $(2x + 5)$  feet. Write a polynomial expression for the area  $A$  of the garden.

#### SOLUTION

The area model for a rectangle is  $A = (\text{width})(\text{length})$ .

$A = (\text{width})(\text{length})$	Area model for a rectangle
$= (x + 6)(2x + 5)$	Substitute $x + 6$ for width and $2x + 5$ for length.
$= 2x^2 + 5x + 12x + 30$	Use FOIL pattern.
$= 2x^2 + 17x + 30$	Combine like terms.

The area  $A$  of the garden can be represented by  $2x^2 + 17x + 30$ .

#### *Exercise for Example 4*

8. Rework Example 4 if the width is  $(x + 3)$  feet and the length is  $(3x + 2)$  feet.

## Practice with Examples

For use with pages 594–601

**GOAL**

Factor trinomials of the form  $x^2 + bx + c$ .

**VOCABULARY**

To factor a trinomial means to write it as the product of two binomials.

To factor  $x^2 + bx + c$ , you need to find numbers  $p$  and  $q$  such that

$$p + q = b \quad \text{and} \quad pq = c.$$

$$x^2 + bx + c = (x + p)(x + q) \quad \text{when} \quad p + q = b \quad \text{and} \quad pq = c$$

**EXAMPLE 1**

### Factoring when $b$ and $c$ Are Positive

Factor  $x^2 + 6x + 8$ .

**SOLUTION**

For this trinomial,  $b = 6$  and  $c = 8$ . You need to find two numbers whose sum is 6 and whose product is 8.

$$\begin{aligned} x^2 + 6x + 8 &= (x + p)(x + q) && \text{Find } p \text{ and } q \text{ when } p + q = 6 \text{ and } pq = 8. \\ &= (x + 4)(x + 2) && p = 4 \text{ and } q = 2 \end{aligned}$$

### Exercises for Example 1

Factor the trinomial.

1.  $x^2 + 5x + 6$

2.  $x^2 + 6x + 5$

3.  $x^2 + 3x + 2$

## Practice with Examples

For use with pages 594–601

### **EXAMPLE 3** Factoring when $b$ Is Negative and $c$ Is Positive

Factor  $x^2 - 5x + 4$ .

#### **SOLUTION**

Because  $b$  is negative and  $c$  is positive, both  $p$  and  $q$  must be negative numbers. Find two numbers whose sum is  $-5$  and whose product is  $4$ .

$$\begin{aligned} x^2 - 5x + 4 &= (x + p)(x + q) && \text{Find } p \text{ and } q \text{ when } p + q = -5 \text{ and } pq = 4. \\ &= (x - 4)(x - 1) && p = -4 \text{ and } q = -1 \end{aligned}$$

#### *Exercises for Example 2*

Factor the trinomial.

4.  $x^2 - 3x + 2$

5.  $x^2 - 7x + 12$

6.  $x^2 - 5x + 6$

### **EXAMPLE 2** Factoring when $b$ and $c$ Are Negative

Factor  $x^2 - 3x - 10$ .

#### **SOLUTION**

For this trinomial,  $b = -3$  and  $c = -10$ . Because  $c$  is negative, you need to find numbers  $p$  and  $q$  with different signs.

$$\begin{aligned} x^2 - 3x - 10 &= (x + p)(x + q) && \text{Find } p \text{ and } q \text{ when } p + q = -3 \text{ and } pq = -10. \\ &= (x + 2)(x - 5) && p = 2 \text{ and } q = -5 \end{aligned}$$

#### *Exercises for Example 3*

Factor the trinomial.

7.  $x^2 - x - 2$

8.  $x^2 - 4x - 12$

9.  $x^2 - 2x - 8$

**Practice with Examples**

For use with pages 594–601

**EXAMPLE 4** *Solving a Quadratic Equation*Solve  $x^2 + 4x = 12$  by factoring.**SOLUTION**

$$x^2 + 4x = 12$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

$$x + 6 = 0 \text{ or } x - 2 = 0$$

$$x + 6 = 0$$

$$x = -6$$

$$x - 2 = 0$$

$$x = 2$$

Write equation.

Write in standard form.

Factor left side. Because  $c$  is negative, you need to find numbers  $p$  and  $q$  with different signs. So,  $p = 6$  and  $q = -2$ .

Use zero-product property.

Set first factor equal to 0.

Solve for  $x$ .

Set second factor equal to 0.

Solve for  $x$ .The solutions are  $-6$  and  $2$ .**Exercises for Example 4****Solve the equation by factoring.**

10.  $x^2 + 8x + 15 = 0$

11.  $x^2 - 8x + 12 = 0$

12.  $x^2 + 3x - 4 = 0$

**Practice with Examples**

For use with pages 646–651

**GOAL** Simplify rational expressions.**VOCABULARY**

A **rational number** is a number that can be written as the quotient of two integers.

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials.

A rational expression is in **simplest form** if its numerator and denominator have no factors in common (other than  $\pm 1$ ).

**EXAMPLE 1** *Simplify a Rational Expression*

Simplify the expression if possible.

a.  $\frac{x^2 - 5}{x}$

b.  $\frac{x^2 - 6x}{3x^2}$

**SOLUTION**

- a. When you simplify rational expressions, you can divide out only factors, not terms. You cannot simplify  $\frac{x^2 - 5}{x}$ . You cannot divide out the common term  $x$ .

b.  $\frac{x^2 - 6x}{3x^2} = \frac{\cancel{x}(x - 6)}{\cancel{x} \cdot 3x}$       You can divide out the common factor  $x$ .

$= \frac{x - 6}{3x}$       Simplify.

**Exercises for Example 1**

Simplify the expression if possible.

1.  $\frac{3x}{4x + x^2}$

2.  $\frac{x^2(x - 7)}{x^3}$

3.  $\frac{x^3 + 3}{x^3}$

**Practice with Examples**

For use with pages 646–651

**EXAMPLE 2** *Recognize Opposite Factors*Simplify  $\frac{x^2 - 6x + 8}{4 - x}$ .**SOLUTION**

$$\frac{x^2 - 6x + 8}{4 - x} = \frac{(x - 2)(x - 4)}{4 - x}$$

Factor numerator and denominator.

$$= \frac{(x - 2)(x - 4)}{-(x - 4)}$$

Factor  $-1$  from  $(4 - x)$ .

$$= \frac{(x - 2)(\cancel{x - 4})}{-(\cancel{x - 4})}$$

Divide out common factor  $(x - 4)$ .

$$= -(x - 2)$$

Simplify.

$$= 2 - x$$

Simplify.

**Exercises for Example 2**

Simplify the expression if possible.

4.  $\frac{x^2 - 8x + 12}{2 - x}$

5.  $\frac{1 - x^2}{x^2 - 3x + 2}$

6.  $\frac{1 - x}{x^2 + 2x - 3}$



**Practice with Examples**

For use with pages 646–651

**EXAMPLE 3** *Divide a Polynomial by a Binomial*Divide  $(4y^2 - 10y - 6)$  by  $(y - 3)$ .**SOLUTION**

$$\frac{4y^2 - 10y - 6}{y - 3}$$

Rewrite the problem as a rational expression.

$$\frac{2(2y + 1)(y - 3)}{y - 3}$$

Factor the numerator.

$$\frac{2(2y + 1)\cancel{(y - 3)}}{\cancel{y - 3}}$$

Divide out the common factor  $(y - 3)$ .

$$4y + 2$$

Simplify the expression.

**Exercise for Example 3**7. Divide  $(6y^2 + 15y - 9)$  by  $(y + 3)$ .

**Practice with Examples**

For use with pages 131–137

**GOAL**

Solve linear equations using addition and subtraction

**VOCABULARY**

Equivalent equations have the same solutions.

Inverse operations are two operations that undo each other, such as addition and subtraction.

Each time you apply a transformation to an equation, you are writing a solution step.

In a linear equation, the variable is raised to the *first* power and does not occur inside a square root symbol, an absolute value symbol, or in a denominator.**EXAMPLE 1****Adding to Each Side**Solve  $y - 7 = -2$ .**SOLUTION**To isolate  $y$ , you need to undo the subtraction by applying the inverse operation of adding 7.

$$y - 7 = -2 \quad \text{Write original equation.}$$

$$y - 7 + 7 = -2 + 7 \quad \text{Add 7 to each side.}$$

$$y = 5 \quad \text{Simplify.}$$

The solution is 5. Check by substituting 5 for  $y$  in the original equation.**Exercises for Example 1**

Solve the equation.

1.  $t - 11 = 4$

2.  $x - 2 = -3$

3.  $5 = d - 8$

**Practice with Examples**

For use with pages 131–137

**EXAMPLE 2** *Subtracting from Each Side*Solve  $q + 4 = -9$ .**SOLUTION**

To isolate  $q$ , you need to undo the addition by applying the inverse operation of subtracting 4.

$$q + 4 = -9 \quad \text{Write original equation.}$$

$$q + 4 - 4 = -9 - 4 \quad \text{Subtract 4 from each side.}$$

$$q = -13 \quad \text{Simplify.}$$

The solution is  $-13$ . Check by substituting  $-13$  for  $q$  in the original equation.

**Exercises for Example 2**

Solve the equation.

4.  $s + 1 = -8$

5.  $-6 + b = 10$

6.  $6 = w + 12$

**EXAMPLE 3** *Simplifying First*Solve  $x - (-3) = 10$ .**SOLUTION**

$$x - (-3) = 10 \quad \text{Write original equation.}$$

$$x + 3 = 10 \quad \text{Simplify.}$$

$$x + 3 - 3 = 10 - 3 \quad \text{Subtract 3 from each side.}$$

$$x = 7 \quad \text{Simplify.}$$

The solution is 7. Check by substituting 7 for  $x$  in the original equation.

**Practice with Examples**

For use with pages 131–137

**Exercises for Example 3**

Solve the equation.

7.  $8 + z = 1$

8.  $7 = k - 2$

9.  $9 = a + (-5)$

**EXAMPLE 4** **Modeling a Real-Life Problem**

The original price of a bicycle was marked down \$20 to a sale price of \$85. What was the original price?

**SOLUTION**

Original price ( $p$ ) – Price reduction (20) = Sale Price (85)

Solve the equation  $p - 20 = 85$ .

$$p - 20 = 85 \quad \text{Write real-life equation.}$$

$$p - 20 + 20 = 85 + 20 \quad \text{Add 20 to each side.}$$

$$p = 105 \quad \text{Simplify.}$$

The original price was \$105. Check this in the statement of the problem.

**Exercise for Example 4**

10. After a sale, the price of a stereo was marked up \$35 to a regular price of \$310. What was the sale price?

**Practice with Examples**

For use with pages 138–143

**GOAL** Solve linear equations using multiplication and division.**VOCABULARY**

Properties of equality are rules of algebra that can be used to transform equations into equivalent equations.

**EXAMPLE 1** *Dividing Each Side of an Equation*Solve  $7n = -35$ .**SOLUTION**

To isolate  $n$ , you need to undo the multiplication by applying the inverse operation of dividing by 7.

$$7n = -35 \quad \text{Write original equation.}$$

$$\frac{7n}{7} = \frac{-35}{7} \quad \text{Divide each side by 7.}$$

$$n = -5 \quad \text{Simplify.}$$

The solution is  $-5$ . Check by substituting  $-5$  for  $n$  in the original equation.

**Exercises for Example 1**

Solve the equation.

1.  $-12x = 6$

2.  $4 = 24y$

3.  $-5z = -35$

**Practice with Examples**

For use with pages 138–143

**EXAMPLE 2** *Multiplying Each Side of an Equation*

Solve  $-\frac{3}{4}t = 9$ .

**SOLUTION**To isolate  $t$ , you need to multiply by the reciprocal of the fraction.

$$-\frac{3}{4}t = 9$$

Write original equation.

$$\left(-\frac{4}{3}\right)\left(-\frac{3}{4}\right)t = \left(-\frac{4}{3}\right)9$$

Multiply each side by  $-\frac{4}{3}$ .

$$t = -12$$

Simplify.

The solution is  $-12$ . Check by substituting  $-12$  for  $t$  in the original equation.**Exercises for Example 2**

Solve the equation.

4.  $\frac{1}{6}c = -2$

5.  $\frac{f}{7} = 3$

6.  $\frac{2}{3}q = 12$

**Practice with Examples**

For use with pages 138–143

**EXAMPLE 3** *Modeling a Real-Life Problem*

Write and solve an equation to find your average speed  $s$  on a plane flight. You flew 525 miles in 1.75 hours.

**SOLUTION**

Verbal Model  $\boxed{\text{Speed of jet}} \cdot \boxed{\text{Time}} = \boxed{\text{Distance}}$

Labels      Speed of jet =  $s$                       (miles per hour)  
                  Time = 1.75                              (hours)  
                  Distance = 525                              (miles)

Algebraic Model       $s(1.75) = 525$       Write algebraic model.  
                                   $\frac{s(1.75)}{1.75} = \frac{525}{1.75}$       Divide each side by 1.75.  
                                   $s = 300$       Simplify.

The speed  $s$  was 300 miles per hour. Check this in the statement of the problem.

**Exercises for Example 3**

7. Write and solve an equation to find your average speed in an airplane if you flew 800 miles in 2.5 hours.
8. Write and solve an equation to find your time in an airplane if you flew 1530 miles at a speed of 340 miles per hour.

**Practice with Examples**

For use with pages 144–149

**GOAL**

Use two or more steps to solve a linear equation

**EXAMPLE 1****Solving a Linear Equation**Solve  $-3x - 4 = 5$ .**SOLUTION**To isolate the variable  $x$ , undo the subtraction and then the multiplication.

$$-3x - 4 = 5 \quad \text{Write original equation.}$$

$$-3x - 4 + 4 = 5 + 4 \quad \text{Add 4 to each side.}$$

$$-3x = 9 \quad \text{Simplify.}$$

$$\frac{-3x}{-3} = \frac{9}{-3} \quad \text{Divide each side by } -3.$$

$$x = -3 \quad \text{Simplify.}$$

The solution is  $-3$ . Check this in the original equation.**Exercises for Example 1**

Solve the equation.

1.  $5y + 8 = -2$

2.  $7 - 6m = 1$

3.  $\frac{x}{4} - 1 = 5$





**Practice with Examples**

For use with pages 144–149

**EXAMPLE 2** *Using the Distributive Property and Combining Like Terms*Solve  $y + 5(y + 3) = 33$ .**SOLUTION**

$$y + 5(y + 3) = 33$$

Write original equation.

$$y + 5y + 15 = 33$$

Use distributive property.

$$6y + 15 = 33$$

Combine like terms.

$$6y + 15 - 15 = 33 - 15$$

Subtract 15 from each side.

$$6y = 18$$

Simplify.

$$\frac{6y}{6} = \frac{18}{6}$$

Divide each side by 6.

$$y = 3$$

Simplify.

The solution is 3. Check this in the original equation.

**Exercises for Example 2**

Solve the equation.

4.  $4x - 8 + x = 2$

5.  $6 - (b + 1) = 9$

6.  $10(z - 2) = 1 + 4$

**Practice with Examples**

For use with pages 144–149

**EXAMPLE 3 Solving a Real-Life Problem**

The sum of the ages of two sisters is 25. The second sister's age is 5 more than three times the first sister's age  $n$ . Find the two ages.

**SOLUTION**

<b>Verbal Model</b>	First sister's age	+	Second sister's age	=	Sum
---------------------	--------------------	---	---------------------	---	-----

<b>Labels</b>	First sister's age = $n$	(years)
	Second sister's age = $3n + 5$	(years)
	Sum = 25	(years)

<b>Algebraic Model</b>	$n + (3n + 5) = 25$	Write real-life equation.
	$4n + 5 = 25$	Combine like terms.
	$4n + 5 - 5 = 25 - 5$	Subtract 5 from each side.
	$4n = 20$	Simplify.
	$\frac{4n}{4} = \frac{20}{4}$	Divide each side by 4.
	$n = 5$	Simplify.

The first sister's age is 5. The second sister's age is  $3(5) + 5 = 20$ .

**Exercises for Example 3**

7. A parking garage charges \$3 plus \$1.50 per hour. You have \$12 to spend for parking. Write and solve an equation to find the number of hours that you can park.
8. As a lifeguard, you earn \$6 per day plus \$2.50 per hour. Write and solve an equation to find how many hours you must work to earn \$16 in one day.

**Practice with Examples**

For use with pages 150–156

**GOAL**

Solve equations that have variables on both sides.

**VOCABULARY**

An identity is an equation that is true for all values of the variable.

**EXAMPLE 1****Collecting Variables on One Side**Solve  $20 - 3x = 2x$ .**SOLUTION**Think of  $20 - 3x$  as  $20 + (-3x)$ . Since  $2x$  is greater than  $-3x$ , collect the  $x$ -terms on the right side.

$$20 - 3x = 2x$$

Write original equation.

$$20 - 3x + 3x = 2x + 3x$$

Add  $3x$  to each side.

$$20 = 5x$$

Simplify.

$$\frac{20}{5} = \frac{5x}{5}$$

Divide each side by 5.

$$4 = x$$

Simplify.

**Exercises for Example 1**

Solve the equation.

1.  $5q = -7q + 6$

2.  $14d - 6 = 17d$

3.  $-y + 7 = -8y$

**Practice with Examples**

For use with pages 150–156

**EXAMPLE 2** *Identifying the Number of Solutions*

a. Solve  $2x + 3 = 2x + 4$ .

b. Solve  $-(t + 5) = -t - 5$ .

**SOLUTION**

a.  $2x + 3 = 2x + 4$  Write original equation.

$2x + 3 - 3 = 2x + 4 - 3$  Subtract 3 from each side.

$2x = 2x + 1$  Simplify.

$0 = 1$  Subtract  $2x$  from each side.

The original equation has *no solution*, because  $0 \neq 1$  for any value of  $x$ .

b.  $-(t + 5) = -t - 5$  Write original equation.

$-t - 5 = -t - 5$  Use distributive property.

$-5 = -5$  Add  $t$  to each side.

All values of  $t$  are solutions, because  $-5 = -5$  is always true.The original equation is an *identity*.**Exercises for Example 2**

Solve the equation.

4.  $9z - 3 = 9z$

5.  $2(f - 7) = 2f - 14$

6.  $n + 3 = -5n$

**Practice with Examples**

For use with pages 150–156

**EXAMPLE 3 Solving Real-Life Problems**

A health club charges nonmembers \$2 per day to swim and \$5 per day for aerobics classes. Members pay a yearly fee of \$200 plus \$3 per day for aerobics classes. Write and solve an equation to find the number of days you must use the club to justify a yearly membership.

**SOLUTION**

Let  $n$  represent the number of days that you use the club. Then find the number of times for which the two plans would cost the same.

$$2n + 5n = 200 + 3n \quad \text{Write equation.}$$

$$7n = 200 + 3n \quad \text{Combine like terms.}$$

$$7n - 3n = 200 + 3n - 3n \quad \text{Subtract } 3n \text{ from each side.}$$

$$4n = 200 \quad \text{Simplify.}$$

$$\frac{4n}{4} = \frac{200}{4} \quad \text{Divide each side by 4.}$$

$$n = 50 \quad \text{Simplify.}$$

You must use the club 50 days to justify a yearly membership.

**Exercises for Example 3**

7. Rework Example 3 if nonmembers pay \$3 per day to swim.
8. Rework Example 3 if members pay a yearly fee of \$220.

**Practice with Examples**

For use with pages 157–162

**GOAL**

Solve more complicated equations that have variables on both sides.

**EXAMPLE 1****Solving a More Complicated Equation**

Solve the equation.

a.  $2(x - 5) + 10 = -(-3x + 2)$

b.  $-3(5x + 1) + 2x = 2(4x - 3)$

c.  $\frac{1}{2}(16 - 6x) = 15 - \frac{1}{3}(9 + 15x)$

**SOLUTION**

a.  $2(x - 5) + 10 = -(-3x + 2)$

$2x - 10 + 10 = 3x - 2$

$2x = 3x - 2$

$-x = -2$

$x = 2$

Write original equation.

Use distributive property.

Combine like terms.

Subtract  $3x$  from each side.Divide each side by  $-1$ .

The solution is 2. Check this in the original equation.

b.  $-3(5x + 1) + 2x = 2(4x - 3)$

$-15x - 3 + 2x = 8x - 6$

$-13x - 3 = 8x - 6$

$-21x - 3 = -6$

$-21x = -3$

$x = \frac{1}{7}$

Write original equation.

Use distributive property.

Combine like terms.

Subtract  $8x$  from each side.

Add 3 to each side.

Divide each side by  $-21$ .The solution is  $\frac{1}{7}$ . Check this in the original equation.

**Practice with Examples**

For use with pages 157–162

$$\begin{array}{ll} \text{c. } \frac{1}{2}(16 - 6x) = 15 - \frac{1}{3}(9 + 15x) & \text{Write original equation.} \\ 8 - 3x = 15 - 3 - 5x & \text{Use distributive property.} \\ 8 - 3x = 12 - 5x & \text{Combine like terms.} \\ 8 + 2x = 12 & \text{Add } 5x \text{ to each side.} \\ 2x = 4 & \text{Subtract 8 from each side.} \\ x = 2 & \text{Divide each side by 2.} \end{array}$$

The solution is 2. Check this in the original equation.

**Exercises for Example 1**

1.  $2(x + 5) + 3x = 3(-2x - 1)$

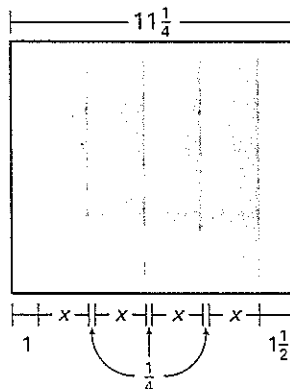
2.  $\frac{3}{4}(8x - 20) = 6x + 12 - 12x$

**EXAMPLE 2****Drawing a Diagram**

The front page of your school newspaper is  $11\frac{1}{4}$  inches wide. The left margin is 1 inch and the right margin is  $1\frac{1}{2}$  inches. The space between the four columns is  $\frac{1}{4}$  inch. Find the width of each column.

**SOLUTION**

The diagram shows that the page is made up of the width of the left margin, the width of the right margin, three spaces between the columns, and the four columns.



**Practice with Examples**

For use with pages 157–162

<b>Verbal Model</b>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Left margin</div> + <div style="border: 1px solid black; padding: 2px; display: inline-block;">Right margin</div> + 3 · <div style="border: 1px solid black; padding: 2px; display: inline-block;">Space between columns</div> + 4 · <div style="border: 1px solid black; padding: 2px; display: inline-block;">Column width</div> = <div style="border: 1px solid black; padding: 2px; display: inline-block;">Page width</div>
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<b>Labels</b>	Left margin = 1	(inch)
	Right margin = $1\frac{1}{2}$	(inches)
	Space between columns = $\frac{1}{4}$	(inch)
	Column width = $x$	(inches)
	Page width = $11\frac{1}{4}$	(inches)

**Algebraic Model**

$$1 + 1\frac{1}{2} + 3\left(\frac{1}{4}\right) + 4x = 11\frac{1}{4}$$

Solving for  $x$ , you find that each column can be 2 inches wide.**Exercise for Example 2**

3. Rework Example 2 if the front page of the newspaper has three columns.



**Practice with Examples**

For use with pages 92–98

**GOAL**

Multiply real numbers using the rules for the sign of a product.

**VOCABULARY**

**Closure property** The product of any two real numbers is a unique real number.  $ab$  is a unique real number.

**Commutative property** The order in which two numbers are multiplied does not change the product.  $ab = ba$

**Associative property** The way three numbers are grouped when multiplying does not change the product.  $(ab)c = a(bc)$

**Identity property** The product of a number and 1 is the number.

$$1 \cdot a = a$$

**Property of zero** The product of a number and 0 is 0.  $0 \cdot a = 0$

**Property of negative 1** The product of a number and  $-1$  is the opposite of the number.  $-1 \cdot a = -a$

**EXAMPLE 1****Multiplying Real Numbers**

Find the product.

a.  $(0.5)(-26)$

b.  $(-1)(-5)(-6)$

c.  $(-4)(6)\left(-\frac{1}{3}\right)$

**SOLUTION**

a.  $(0.5)(-26) = -13$       One negative factor

b.  $(-1)(-5)(-6) = -30$       Three negative factors

c.  $(-4)(6)\left(-\frac{1}{3}\right) = 8$       Two negative factors

**Exercises for Example 1**

Find the product.

1.  $(-2)(3)$

2.  $(-7)(-1)$

3.  $(10)(-2)$

4.  $(-12)(0.5)(-3)$

5.  $(-4)(-2)(-5)$

6.  $(6)(-6)(2)$

**Practice with Examples**

For use with pages 92–98

**EXAMPLE 2** *Simplifying Variable Expressions*

Simplify the expression.

a.  $(-2)(-7x)$

b.  $-(y)^2$

**SOLUTION**

a.  $(-2)(-7x) = 14x$

Two negative factors, so product is positive

b.  $-(y)^2 = (-1)(y^2) = -y^2$

One negative factor, so product is negative

*Exercises for Example 2*

Simplify the variable expression.

7.  $(5)(-w)$

8.  $8(-t)(-t)$

9.  $(-7)(-y)(-y)$

10.  $-\frac{1}{3}(6x)$

11.  $-4(a)(-a)(-a)$

12.  $-\frac{3}{5}(-s)(10s)$

**EXAMPLE 3** *Evaluating a Variable Expression*Evaluate the expression  $(-12 \cdot x)(-3)$  when  $x = -2$ .**SOLUTION**

$(-12 \cdot x)(-3) = 36x$

Simplify expression first.

$= 36(-2)$

Substitute  $-2$  for  $x$ .

$= -72$

Simplify.

*Exercises for Example 3*

Evaluate the expression.

13.  $-15x$  when  $x = 3$

14.  $2p^2$  when  $p = -1$

15.  $(-4m^2)(5m)$  when  $m = -2$

16.  $k^3$  when  $k = -3$

## ***Practice with Examples***

For use with pages 92–98

### **EXAMPLE 4** *Using Multiplication in Real Life*

To promote its grand opening, a record store advertises compact discs for \$10. The store loses \$2.50 on each compact disc it sells. How much money will the store lose on its grand opening sale if it sells 256 discs?

#### **SOLUTION**

Multiply the number of discs sold by the loss per disc to find the total loss.

$$(256)(-2.50) = -640$$

The store loses \$640 on its grand opening sale.

#### ***Exercises for Example 4***

17. Rework Example 4 if the store loses \$1.50 on each disc.

18. Rework Example 4 if the store sells 185 discs.

**Practice with Examples**

For use with pages 92–98

**EXAMPLE 4****Using Multiplication in Real Life**

To promote its grand opening, a record store advertises compact discs for \$10. The store loses \$2.50 on each compact disc it sells. How much money will the store lose on its grand opening sale if it sells 256 discs?

**SOLUTION**

Multiply the number of discs sold by the loss per disc to find the total loss.

$$(256)(-2.50) = -640$$

The store loses \$640 on its grand opening sale.

**Exercises for Example 4**

17. Rework Example 4 if the store loses \$1.50 on each disc.
18. Rework Example 4 if the store sells 185 discs.

# Practice with Examples

For use with pages 99–106

**GOAL** Use the distributive property.

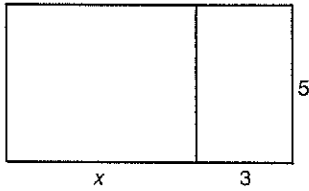
**VOCABULARY**

**Distributive property:** the product of  $a$  and  $b + c$  or of  $a$  and  $b - c$ :

$$\begin{aligned} a(b + c) &= ab + ac & a(b - c) &= ab - ac \\ (b + c)a &= ba + ca & (b - c)a &= ba - ca \end{aligned}$$

**EXAMPLE 1** Use an Area Model

Find the area of a rectangle whose width is 5 and whose length is  $x + 3$ .

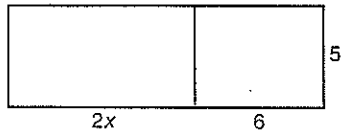


**SOLUTION**

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= (x + 3)5 \\ &= 5x + 15 \end{aligned}$$

**Exercises for Example 1**

- Write two expressions for the area of the rectangle below.



- Write an algebraic statement that shows that the two expressions from Exercise 1 are equal.

**Practice with Examples**

For use with pages 99–106

**EXAMPLE 2** *Use the Distributive Property with Addition*

Use the distributive property to rewrite the expression without parentheses.

a.  $-4(2x + 1)$

b.  $(3x + 3)5$

**SOLUTION**

a.  $-8x - 4$

b.  $15x + 15$

*Exercises for Example 2*

Use the distributive property to rewrite the expression without parentheses.

3.  $3(2x + 6)$

4.  $-5(6x + 1)$

5.  $-1(x + 8)$

**EXAMPLE 3** *Use the Distributive Property with Subtraction*

Use the distributive property to rewrite the expression without parentheses.

a.  $4(x - 6)$

b.  $-3(2x - 8)$

**SOLUTION**

a.  $4x - 24$

b.  $-6x + 24$

*Exercises for Example 3*

Use the distributive property to rewrite the expression without parentheses.

6.  $-3(2x - 9)$

7.  $(4x - 3)6$

8.  $\frac{1}{2}(6x - 10)$

**Practice with Examples**

For use with pages 99–106

**EXAMPLE 4** *Mental Math Calculations*Use the distributive property to mentally calculate  $23 \times 6$ .**SOLUTION**

$$23 \times 6 = (20 + 3)6 = 20 \cdot 6 + 3 \cdot 6 = 120 + 18 = 138$$

**Exercises for Example 4**

Use the distributive property to mentally calculate the expression.

9.  $16 \times 3$

10.  $2 \times 38$

11.  $24 \times 4$

**Practice with Examples**

For use with pages 298–304

**GOAL**

Write and use a linear equation to solve a real-life problem.

**VOCABULARY**

A **linear model** is a linear function that is used to model a real-life situation.

A **rate of change** compares two quantities that are changing.

**EXAMPLE 1****Write a Linear Model**

In 1995 you had an investment worth \$1000. It decreased in value by about \$50 per year. Write a linear model for the value of your investment  $y$ . Let  $t = 0$  represent 1995.

**SOLUTION**

The rate of decrease is \$50 per year, so the slope is  $m = -50$ . The year 1995 is represented by  $t = 0$ . Therefore, the point  $(t_1, y_1)$  is  $(0, 1000)$ .

$$y - y_1 = m(t - t_1) \quad \text{Write the point-slope form.}$$

$$y - 1000 = -50(t - 0) \quad \text{Substitute values.}$$

$$y - 1000 = -50t \quad \text{Use the distributive property}$$

$$y = -50t + 1000 \quad \text{Add 1000 to each side.}$$

**Exercises for Example 1**

1. You begin a hiking trail at 8:00 A.M. and hike at a rate of 3 miles per hour. Write a linear model for the number of miles hiked  $y$ . Let  $t = 0$  represent 8:00 A.M.
2. You make an initial investment of \$500 in 1995. It increases in value by about \$100 per year. Write a linear model for the value of the investment in year  $t$ . Let  $t = 0$  represent 1995.



***Practice with Examples***

For use with pages 298–304

**EXAMPLE 2*****Use a Linear Model to Predict***

Use the linear model in Example 1 to predict the value of your investment in 2003.

$$y = -50t + 1000$$

Write the linear model.

$$y = -50(8) + 1000$$

Substitute 8 for  $t$ .

$$y = -400 + 1000$$

Simplify.

$$y = 600$$

Solve for  $y$ .

In 2003 your investment will be worth \$600.

***Exercises for Example 2***

- Use the linear model you wrote in Exercise 1 to predict how many miles you will have hiked at 10 A.M. if you continue at the same rate.
- Use the linear model you wrote in Exercise 2 to predict the value of your investment in 2001 if it increases at the same rate.

**Practice with Examples**

For use with pages 298–304

**EXAMPLE 3** *Write and Use a Linear Model*

You are buying carrots and peas for dinner. The carrots cost \$1.50 per pound and the peas cost \$.75. You have \$4.50 to spend.

- Write an equation that models the different amounts (in pounds) of carrots and peas you can buy.
- Use the model to complete the table that illustrates several different amounts of carrots and peas you can buy.

<i>Carrots (lb), x</i>	0	1	2	3
<i>Peas (lb), y</i>	?	?	?	?

**SOLUTION**

- Let the amount of carrots (in pounds) be  $x$  and the amount of peas (in pounds) be  $y$ .

<b>Verbal Model</b>	Price of carrots	·	Weight of carrots	+	Price of peas	·	Weight of peas	=	Total cost
---------------------	------------------	---	-------------------	---	---------------	---	----------------	---	------------

<b>Labels</b>	Price of carrots = 1.5	(dollars per pound)
	Weight of carrots = $x$	(pounds)
	Price of peas = 0.75	(dollars per pound)
	Weight of peas = $y$	(pounds)
	Total cost = 4.50	(dollars)

**Algebraic Model**       $1.5x + 0.75y = 4.50$

- Complete the table by substituting the given values of  $x$  into the equation  $1.5x + 0.75y = 4.50$  to find  $y$ .

<i>Carrots (lb), x</i>	0	1	2	3
<i>Peas (lb), y</i>	6	4	2	?

**Exercise for Example 3**

- You are buying jeans and shirts. You have \$120. Jeans cost \$40 and shirts cost \$20. Write an equation that models the different amounts of jeans and shirts you can afford to buy. Use a table to show the different combinations of jeans and shirts you can buy.

# Practice with Examples

For use with pages 389–395

**GOAL** Estimate the solution of a system of linear equations by graphing.

### VOCABULARY

Two or more linear equations in the same variables form a **system of linear equations** or simply a **linear system**.

A **solution of a system of linear equations** in two variables is a pair of numbers  $a$  and  $b$  for which  $x = a$  and  $y = b$  make each equation a true statement.

The point  $(a, b)$  lies on the graph of each equation and is the **point of intersection** of the graphs.

### EXAMPLE 1 Graph and Check a Linear System

Solve the linear system graphically. Check the solution algebraically.

$$-3x + y = -7 \quad \text{Equation 1}$$

$$2x + 2y = 10 \quad \text{Equation 2}$$

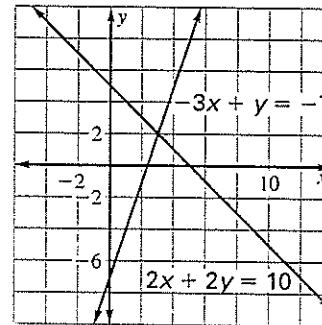
#### SOLUTION

Write each equation in slope-intercept form.

$$y = 3x - 7 \quad \text{Slope: 3, } y\text{-intercept: } -7$$

$$y = -x + 5 \quad \text{Slope: } -1, y\text{-intercept: } 5$$

Graph each equation. The lines appear to intersect at  $(3, 2)$ .



To check  $(3, 2)$  as a solution algebraically, substitute 3 for  $x$  and 2 for  $y$  in each original equation.

EQUATION 1	EQUATION 2
$-3x + y = -7$	$2x + 2y = 10$
$-3(3) + 2 \stackrel{?}{=} -7$	$2(3) + 2(2) \stackrel{?}{=} 10$
$-7 = -7$	$10 = 10$

Because  $(3, 2)$  is a solution of each equation in the linear system, it is a solution of the linear system.

**Practice with Examples**

For use with pages 389–395

**Exercises for Example 1**

Estimate the solution of the linear system graphically. Then check the solution algebraically.

$$\begin{aligned} 1. \quad & y = -x + 5 \\ & y = x + 1 \end{aligned}$$

$$\begin{aligned} 2. \quad & 2x - y = 2 \\ & x = 4 \end{aligned}$$

$$\begin{aligned} 3. \quad & 2x + y = 2 \\ & x - y = 4 \end{aligned}$$

**EXAMPLE 2****Using a Linear System to Model a Real-Life Problem**

Tickets for the theater are \$5 for the balcony and \$10 for the orchestra. If 600 tickets were sold and the total receipts were \$4750, how many tickets of each type were sold?

**SOLUTION**

Verbal Model	Number of balcony tickets	+	Number of orchestra tickets	=	Total number of tickets
	Price of balcony tickets	·	Number of balcony tickets	+	
	Price of orchestra tickets	·	Number of orchestra tickets	=	Total receipts

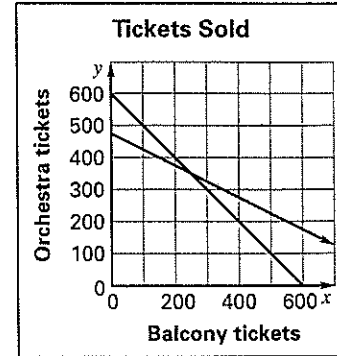
*(continued)*

**Practice with Examples**

For use with pages 389–395

<b>Labels</b>	Price of balcony tickets = 5	(dollars)
	Number of balcony tickets = $x$	(tickets)
	Price of orchestra tickets = 10	(dollars)
	Number of orchestra tickets = $y$	(tickets)
	Total number of tickets = 600	(tickets)
	Total receipts = 4750	(dollars)

<b>Algebraic</b>	$x + y = 600$	Equation 1 (tickets)
<b>Model</b>	$5x + 10y = 4750$	Equation 2 (receipts)



Use the graph-and-check method to solve the system.

The point of intersection of the two lines appears to be (250, 350).

According to the model, 250 tickets for the balcony and 350 tickets for the orchestra were sold.

**Exercises for Example 2**

4. Algebraically check the solution for Example 2.

5. Rework Example 2 if 800 tickets were sold.

6. Rework Example 2 if total receipts were \$3500.

## Practice with Examples

For use with pages 396–401

**GOAL** Solve a linear system by substitution.

### **EXAMPLE 1** *The Substitution Method*

Solve the linear system.  $x + y = 1$  Equation 1  
 $2x - 3y = 12$  Equation 2

#### **SOLUTION**

Solve for  $y$  in Equation 1.

$$y = -x + 1 \quad \text{Revised Equation 1}$$

Substitute  $-x + 1$  for  $y$  in Equation 2 and solve for  $x$ .

$2x - 3y = 12$	Write Equation 2.
$2x - 3(-x + 1) = 12$	Substitute $-x + 1$ for $y$ .
$2x + 3x - 3 = 12$	Distribute the $-3$ .
$5x - 3 = 12$	Combine like terms.
$5x = 15$	Add 3 to each side.
$x = 3$	Divide each side by 5.

To find the value of  $y$ , substitute 3 for  $x$  in the revised Equation 1.

$y = -x + 1$	Write revised Equation 1.
$y = -3 + 1$	Substitute 3 for $x$ .
$y = -2$	Solve for $y$ .

The solution is  $(3, -2)$ .

**Practice with Examples**

For use with pages 396–401

**Exercises for Example 1**

Use the substitution method to solve the linear system.

1.  $x + 2y = -5$

$4x - 3y = 2$

2.  $3x - 2y = 4$

$x + 3y = 5$

3.  $3x + y = -2$

$x + 3y = 2$

**EXAMPLE 2****Writing and Using a Linear System**

An investor bought 225 shares of stock, stock A at \$50 per share and stock B at \$75 per share. If \$13,750 worth of stock was purchased, how many shares of each kind did the investor buy?

**SOLUTION**

Verbal Model	Amount of stock A	+	Amount of stock B	=	Total amount of stock
--------------	-------------------	---	-------------------	---	-----------------------

Price of stock A	·	Amount of stock A	+	Price of stock B	·	Amount of stock B	=	Total investment
------------------	---	-------------------	---	------------------	---	-------------------	---	------------------

Labels	Amount of stock A = $x$	(shares)
--------	-------------------------	----------

Amount of stock B = $y$	(shares)
-------------------------	----------

Total amount of stock = 225	(shares)
-----------------------------	----------

Price of stock A = 50	(dollars per share)
-----------------------	---------------------

Price of stock B = 75	(dollars per share)
-----------------------	---------------------

Total investment = 13,750	(dollars)
---------------------------	-----------

*(continued)*

## ***Practice with Examples***

For use with pages 396–401

<b>Algebraic</b>	$x + y = 225$	Equation 1 (shares)
<b>Model</b>	$50x + 75y = 13,750$	Equation 2 (dollars)

Solve for  $y$  in Equation 1.

$y = -x + 225$	Revised Equation 1
----------------	--------------------

Substitute  $-x + 225$  for  $y$  in Equation 2 and solve for  $x$ .

$50x + 75y = 13,750$	Write Equation 2.
$50x + 75(-x + 225) = 13,750$	Substitute $-x + 225$ for $y$ .
$50x - 75x + 16,875 = 13,750$	Distribute the 75.
$-25x = -3125$	Simplify.
$x = 125$	Solve for $x$ .

To find the value of  $y$ , substitute 125 for  $x$  in the revised Equation 1.

$y = -x + 225$	Write revised Equation 1.
$y = -125 + 225$	Substitute 125 for $x$ .
$y = 100$	Solve for $y$ .

The solution is (125, 100).

The investor bought 125 shares of stock A and 100 shares of stock B.

### ***Exercises for Example 2***

4. Rework Example 2 if the investor bought 200 shares of stock.

5. Rework Example 2 if \$16,250 worth of stock was purchased.



**Practice with Examples**

For use with pages 402–408

**GOAL**

Solve a system of linear equations by linear combinations.

**VOCABULARY**

A **linear combination** of two equations is an equation obtained by  
 (1) multiplying one or both equations by a constant if necessary and  
 (2) adding the resulting equations.

**EXAMPLE 1****Multiply Then Add**Solve the linear system.  $4x - 3y = 11$  Equation 1

$3x + 2y = -13$  Equation 2

**SOLUTION**

The equations are arranged with like terms in columns. You can get the coefficients of  $y$  to be opposites by multiplying the first equation by 2 and the second equation by 3.

$$\begin{array}{rcl}
 4x - 3y = 11 & \text{Multiply by 2.} & 8x - 6y = 22 \\
 3x + 2y = -13 & \text{Multiply by 3.} & \underline{9x + 6y = -39} \\
 & & 17x = -17 \quad \text{Add the equations.} \\
 & & x = -1 \quad \text{Solve for } x.
 \end{array}$$

Substitute  $-1$  for  $x$  in the second equation and solve for  $y$ .

$3x + 2y = -13$  Write Equation 2.

$3(-1) + 2y = -13$  Substitute  $-1$  for  $x$ .

$-3 + 2y = -13$  Simplify.

$y = -5$  Solve for  $y$ .

The solution is  $(-1, -5)$ .**Exercises for Example 1**

Use linear combinations to solve the system of linear equations.

1.  $5x + 2y = 1$

$-5x + 2y = 1$

2.  $-7x + 2y = 4$

$7x + 2y = 4$

**Practice with Examples**

For use with pages 402–408

Use linear combinations to solve the system of linear equations.

3.  $4x - 3y = 9$

$x + 3y = 6$

4.  $x + 2y = 5$

$3x - 2y = 7$

5.  $x + y = 1$

$2x - 3y = 12$

6.  $x - y = -4$

$x + 2y = 5$

**EXAMPLE 2** *Solve by Linear Combinations*Solve the linear system.  $2x + 4y = 10$  Equation 1

$3y = 12 - 2x$  Equation 2

**SOLUTION**

Arrange the equations with like terms in columns.

$2x + 4y = 10$

Write Equation 1.

$2x + 3y = 12$

Rearrange Equation 2.

Multiply Equation 2 by  $-1$  to get the coefficients of  $x$  to be opposites.

$2x + 4y = 10$

$2x + 4y = 10$

$2x + 3y = 12$

Multiply by  $-1$ .  $-2x - 3y = -12$

Add the equations.  $y = -2$

Substitute  $-2$  for  $y$  into either equation and solve for  $x$ .

$2x + 4y = 10$

Write Equation 1.

$2x + 4(-2) = 10$

Substitute  $-2$  for  $y$ .

$2x - 8 = 10$

Multiply.

$2x = 18$

Add 8 to both sides.

$x = 9$

Solve for  $x$ .The solution is  $(9, -2)$ .*(continued)*

**Practice with Examples**

For use with pages 402–408

Check the solution in each of the original equations.

First check the solution in Equation 1.

$$2x + 4y = 10 \quad \text{Write Equation 1.}$$

$$2(9) + 4(-2) \stackrel{?}{=} 10 \quad \text{Substitute 9 for } x \text{ and } -2 \text{ for } y.$$

$$18 - 8 \stackrel{?}{=} 10 \quad \text{Multiply.}$$

$$10 = 10 \quad \text{Subtract.}$$

Then check the solution in Equation 2.

$$2x + 3y = 12 \quad \text{Write Equation 2.}$$

$$2(9) + 3(-2) \stackrel{?}{=} 12 \quad \text{Substitute 9 for } x \text{ and } -2 \text{ for } y.$$

$$18 - 6 \stackrel{?}{=} 12 \quad \text{Multiply.}$$

$$12 = 12 \quad \text{Subtract.}$$

**Exercises for Example 2****Solve the linear systems. Then check your solutions.**

7.  $3x + 4y = 12$

$2x + 4y = 10$

8.  $6x + 5y = 10$

$6x - 2y = 3$

9.  $-4x - 3y = 8$

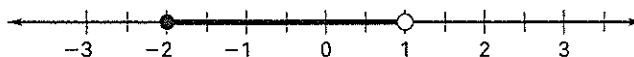
$-4x + 2y = -2$

**Practice with Examples**

For use with pages 342–347

**GOAL**Solve and graph compound inequalities involving *and*.**VOCABULARY**A compound inequality consists of two inequalities connected by *and* or *or*.**EXAMPLE 1****Writing Compound Inequalities with And**Write an inequality that represents all real numbers that are greater than or equal to  $-2$  *and* less than  $1$ . Graph the inequality.**SOLUTION**

$$-2 \leq x < 1$$

**Exercises for Example 1**

Write an inequality that represents the statement and graph the inequality.

- $x$  is greater than  $-4$  *and* less than or equal to  $-2$ .
- $x$  is greater than  $-3$  *and* less than  $-1$ .

**EXAMPLE 2****Compound Inequalities in Real Life**

In 1985, a real estate property was sold for \$145,000. The property was sold again in 1999 for \$211,000. Write a compound inequality that represents the different values that the property was worth between 1985 and 1999.

**SOLUTION**Use the variable  $v$  to represent the property value. Write a compound inequality to represent the different property values.

$$145,000 \leq v \leq 211,000$$

**Practice with Examples**

For use with pages 342–347

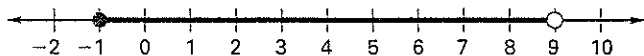
**Exercise for Example 2**

3. Rework Example 2 if the property was sold in 1985 for \$172,000 and was sold again in 1999 for \$226,000.

**EXAMPLE 3****Solve Compound Inequalities with And**Solve  $-4 \leq x - 3 < 6$ . Then graph.**SOLUTION**Isolate the variable  $x$  between the two inequality symbols.

$$-4 \leq x - 3 < 6 \quad \text{Write original inequality.}$$

$$-1 \leq x < 9 \quad \text{Add 3 to each expression.}$$

The solution is all real numbers greater than or equal to  $-1$  and less than  $9$ .**Exercises for Example 3**

Solve the inequality and graph the solution.

4.  $5 \leq x + 7 \leq 9$

5.  $-6 < x - 4 \leq 7$

6.  $0 \leq x - 3 < 10$

## Practice with Examples

For use with pages 342–347

### EXAMPLE 4 Solve Multi-Step Compound Inequalities

Solve  $-9 \leq -4x - 5 < 3$ . Graph the solution.

#### SOLUTION

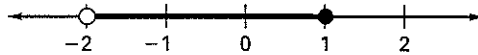
Isolate the variable  $x$  between the two inequality symbols.

$$-9 \leq -4x - 5 < 3 \quad \text{Write original inequality.}$$

$$-4 \leq -4x < 8 \quad \text{Add 5 to each expression.}$$

$$1 \geq x > -2 \quad \text{Divide each expression by } -4 \text{ and} \\ \text{reverse both inequality symbols.}$$

The solution is all real numbers that are less than or equal to 1 *and* greater than  $-2$ .



#### Exercises for Example 4

Solve the inequality and graph the solution.

7.  $-3 < 2x + 1 \leq 7$

8.  $-9 < -3 + 2x < -5$

9.  $2 \leq -3x + 8 < 17$

# Practice with Examples

For use with pages 348–353

**GOAL**

Solve and graph compound inequalities involving *or*.

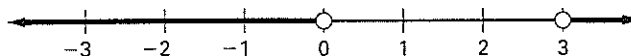
**EXAMPLE 1**

### Writing Compound Inequalities with Or

Write an inequality that represents all real numbers that are less than 0 *or* greater than 3. Graph the inequality.

**SOLUTION**

$$x < 0 \text{ or } x > 3$$



### Exercises for Example 1

Write an inequality that represents the statement and graph the inequality.

- All real numbers less than  $-4$  *or* greater than or equal to  $-2$
- All real numbers greater than  $3$  *or* less than  $-1$

**EXAMPLE 2**

### Solving a Multi-Step Inequality with Or

Solve  $5x + 1 < -4$  *or*  $6x - 2 \geq 10$ . Graph the solution.

**SOLUTION**

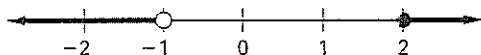
You can solve each part separately.

$$5x + 1 < -4 \quad \text{or} \quad 6x - 2 \geq 10$$

$$5x < -5 \quad \text{or} \quad 6x \geq 12$$

$$x < -1 \quad \text{or} \quad x \geq 2$$

The solution is all real numbers that are less than  $-1$  *or* greater than or equal to  $2$ .



## Practice with Examples

For use with pages 348–353

### Exercises for Example 2

Solve the inequality and graph the solution.

3.  $2x - 3 < 5$  or  $3x + 1 \geq 16$

4.  $-4x + 2 < 6$  or  $2x \leq -6$

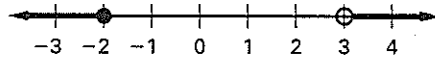
### EXAMPLE 3 Solving a Compound Inequality with Or

Solve  $x - 1 \leq -3$  or  $x + 3 > 6$ . Graph the solution.

#### SOLUTION

$$x - 1 \leq -3 \quad \text{or} \quad x + 3 > 6$$

$$x \leq -2 \quad \text{or} \quad x > 3$$



### Exercises for Example 3

Solve the inequality and graph the solution.

5.  $x + 2 < -4$  or  $x - 3 \geq 0$

6.  $x + 1 < 1$  or  $x - 4 > 1$

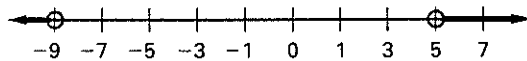


**Practice with Examples**

For use with pages 348–353

**EXAMPLE 4** *Reverse Both Inequalities*Solve  $-x + 3 < -2$  or  $-4 - x > 5$ . Graph the solution.**SOLUTION**

$$\begin{array}{rcl}
 -x + 3 < -2 & \text{or} & -4 - x > 5 \\
 -3 - x + 3 < -2 - 3 & \text{or} & -4 - x + 4 > 5 + 4 \\
 -x < -5 & \text{or} & -x > 9 \\
 (-1)(-x) > -5(-1) & \text{or} & (-1)(-x) < 9(-1) \\
 x > 5 & \text{or} & x < -9
 \end{array}$$

**Exercises for Example 4**

Solve the inequality and graph the solution.

7.  $-x + 5 < -1$  or  $-x - 3 > 7$

8.  $-x - 1 \leq 2$  or  $-x + 2 > 7$

**Practice with Examples**

For use with pages 354–360

**GOAL**

Solve absolute-value equations in one variable.

**VOCABULARY**An absolute-value equation is an equation of the form  $|ax + b| = c$ .**EXAMPLE 1****Solving an Absolute-Value Equation**

a. Solve  $|x| = 4$ .

b. Solve  $|x| = -5$ .

**SOLUTION**

a.  $|x| = 4$

$x = 4$  or  $x = -4$

b.  $|x| = -5$

The absolute value of a number is never negative. The equation  $|x| = -5$  has no solution.**Exercises for Example 1**

Solve the equation.

1.  $|x| = 8$

2.  $|x| = -2$

3.  $|x| = 6$

**EXAMPLE 2****Solving an Absolute-Value Equation**

Solve  $|x - 3| = 4$ .

**SOLUTION**

$|x - 3| = 4$

The expression  $x - 3$  can equal 4 or  $-4$ .

$x - 3 = 4$  or  $x - 3 = -4$

$x = 7$  or  $x = -1$

**Exercises for Example 2**

Solve the absolute-value equation.

4.  $|x + 6| = 8$

5.  $|x - 1| = 8$

6.  $|x + 3| = 11$

**Practice with Examples**

For use with pages 354–360

**EXAMPLE 3** *Solving an Absolute-Value Equation*Solve  $|4x + 2| = 18$ .**SOLUTION**Because  $|4x + 2| = 18$ , the expression  $4x + 2$  can be equal to 18 or  $-18$ . $4x + 2$  IS POSITIVE

$$|4x + 2| = 18$$

$$4x + 2 = +18$$

$$4x = 16$$

$$x = 4$$

 $4x + 2$  IS NEGATIVE

$$|4x + 2| = 18$$

$$4x + 2 = -18$$

$$4x = -20$$

$$x = -5$$

The equation has two solutions: 4 and  $-5$ .**Exercises for Example 3**

Solve the equation.

7.  $|2x + 1| = 15$

8.  $|2x - 6| = 8$

9.  $|2x - 3| = 9$

**Practice with Examples**

For use with pages 354–360

**EXAMPLE 4** *Write an Absolute-Value Equation*

Write an absolute-value equation that has 4 and 8 as its solutions.

**SOLUTION**

The midpoint between 4 and 8 is 6.

The distance of the midpoint from the solutions is 2.

$$|x - 6| = 2$$

↑
↑  
 Midpoint      Distance

Answer:  $|x - 6| = 2$ **Exercises for Example 4**10. Write an absolute-value equation that has  $-2$  and  $6$  as its solutions.

**Practice with Examples**

For use with pages 323–328

**GOAL**

Solve and graph one-step inequalities in one variable using addition or subtraction.

**VOCABULARY**

The graph of an inequality in one variable is the set of points on a number line that represent all solutions of the inequality.

Equivalent inequalities are inequalities that have the same solutions.

Addition property of inequality: For all real numbers

 $a, b,$  and  $c$ : If  $a > b$ , then  $a + c > b + c$ .If  $a < b$ , then  $a + c < b + c$ .

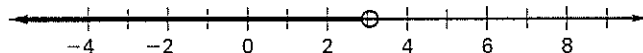
Subtraction property of inequality: For all real numbers

 $a, b,$  and  $c$ : If  $a > b$ , then  $a - c > b - c$ .If  $a < b$ , then  $a - c < b - c$ .**EXAMPLE 1****Graphing an Inequality in One Variable**

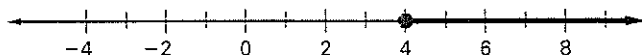
- Graph the inequality  $3 > x$ .
- Graph the inequality  $x \geq 4$ .

**SOLUTION**

- Notice that  $3 > x$  gives the same information as  $x < 3$ . Use an open dot for the inequality symbol  $<$  or  $>$ .



- Use a closed dot for the inequality symbol  $\leq$  or  $\geq$ .

**Exercises for Example 1**

Graph the inequality.

- $x \leq -1$
- $x \geq 0$
- $x < 0$

## Practice with Examples

For use with pages 323–328

### EXAMPLE 2 Using Addition to Solve an Inequality

Solve  $x - 7 > -6$ . Graph the solution.

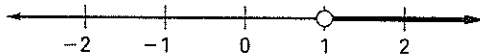
#### SOLUTION

$$x - 7 > -6 \quad \text{Write original inequality.}$$

$$x - 7 + 7 > -6 + 7 \quad \text{Add 7 to each side.}$$

$$x > 1 \quad \text{Simplify.}$$

The solution is all real numbers greater than 1. Check several numbers that are greater than 1 in the original inequality.



#### Exercises for Example 2

Solve the inequality and graph its solution.

4.  $x - 5 < -9$

5.  $a - 3 \leq 0$

6.  $t - 1 < -7$



**Practice with Examples**

For use with pages 323–328

**EXAMPLE 3** *Use Subtraction to Solve an Inequality*Solve  $x + 2 < 7$ . Graph the solution.**SOLUTION**

$$x + 2 < 7$$

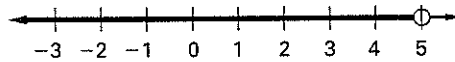
Write original inequality.

$$x + 2 - 2 < 7 - 2$$

Subtract 2 from each side.

$$x < 5$$

Simplify.

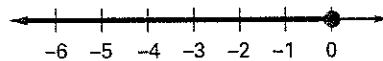
**Exercises for Example 3**

Solve the inequality and graph its solution.

7.  $4 > y + 2$

8.  $x + 3 \leq 0$

9.  $k + 4 > 2$

**EXAMPLE 4** *Write and Graph an Inequality in One Variable*At sea level, water freezes at or below  $0^{\circ}\text{C}$ . Write and graph an inequality that describes the freezing temperature.**SOLUTION**The water temperature is less than or equal to  $0^{\circ}\text{C}$ .  
In symbols,  $x \leq 0$ .**Exercise for Example 4**10. At sea level, ice melts at or above  $32^{\circ}\text{F}$ . Write and graph an inequality that describes the melting temperature.

**Practice with Examples**

For use with pages 329–335

**GOAL**

Solve and graph one-step inequalities in one variable using multiplication or division.

**Multiplication property of inequality:** For all real numbers  $a$  and  $b$ , andfor  $c > 0$ If  $a > b$ , then  $ac > bc$ .If  $a < b$ , then  $ac < bc$ .for  $c < 0$ If  $a > b$ , then  $ac < bc$ .If  $a < b$ , then  $ac > bc$ .**Division property of inequality:** For all real numbers  $a$  and  $b$ , andfor  $c > 0$ If  $a > b$ , then  $\frac{a}{c} > \frac{b}{c}$ .If  $a < b$ , then  $\frac{a}{c} < \frac{b}{c}$ .for  $c < 0$ If  $a > b$ , then  $\frac{a}{c} < \frac{b}{c}$ .If  $a < b$ , then  $\frac{a}{c} > \frac{b}{c}$ .**EXAMPLE 1****Multiply by a Positive Number**

a. Solve  $\frac{x}{3} > 5$ .

b. Solve  $\frac{x}{2} \leq -4$ .

**SOLUTION**

a.  $\frac{x}{3} > 5$

$3 \cdot \frac{x}{3} > 3 \cdot 5$

$x > 15$

b.  $\frac{x}{2} \leq -4$

$2 \cdot \frac{x}{2} \leq 2 \cdot (-4)$

$x \leq -8$

**Exercises for Example 1**

Solve the inequality and graph its solution.

1.  $\frac{x}{4} < -2$

2.  $\frac{t}{2} > 3$

3.  $\frac{b}{5} \geq -3$



**Practice with Examples**

For use with pages 329–335

**EXAMPLE 2****Divide by a Positive Number**

a. Solve  $2x < -4$ .

b. Solve  $3x \geq 15$ .

**SOLUTION**

a.  $2x < -4$

$$\frac{2x}{2} < \frac{-4}{2}$$

$$x < -2$$

b.  $3x \geq 15$

$$\frac{3x}{3} \geq \frac{15}{3}$$

$$x \geq 5$$

**Exercises for Example 2**

Solve the inequality and graph its solution.

4.  $6a > 36$

5.  $4x \leq -16$

6.  $7y \leq -21$

**Practice with Examples**

For use with pages 329–335

**EXAMPLE 3** *Multiply by a Negative Number*

a. Solve  $-\frac{n}{2} < 5$ .

b. Solve  $-\frac{t}{3} \geq -1$ .

**SOLUTION**

When you multiply each side of an inequality by a negative number, you must change the direction of the inequality symbol.

a.  $-\frac{n}{2} < 5$

$$-2 \cdot -\frac{n}{2} > -2 \cdot 5$$

$$n > -10$$

b.  $-\frac{t}{3} \geq -1$

$$-3 \cdot -\frac{t}{3} \leq -3 \cdot (-1)$$

$$t \leq 3$$

**Exercises for Example 3**

Solve the inequality and graph its solution.

7.  $-\frac{1}{2}x > 2$

8.  $-\frac{y}{5} < -2$

9.  $-\frac{z}{2} \leq 3$

**EXAMPLE 4** *Divide by a Negative Number*

a. Solve  $-4n > 20$ .

b. Solve  $-6n < -24$ .

**SOLUTION**

When you divide each side of an inequality by a negative number, you must change the direction of the inequality symbol.

a.  $-4n > 20$

$$\frac{-4n}{-4} < \frac{20}{-4}$$

$$n < -5$$

b.  $-6n < -24$

$$\frac{-6n}{-6} > \frac{-24}{-6}$$

$$n > 4$$

**Exercises for Example 4**

Solve the inequality and graph its solution.

10.  $-2n \leq 12$

11.  $-8p > 32$

12.  $-9q \geq -27$

# Solving Systems of Linear Inequalities

(pp. 67–69)

One way to solve a system of inequalities is to graph each inequality. The graph of the system is the intersection of these graphs.

## 1. Graphing Systems of Two Linear Inequalities

**Vocabulary**

**System of linear inequalities** Two or more linear inequalities in the same variables; also called a *system of inequalities*.

**EXAMPLE**

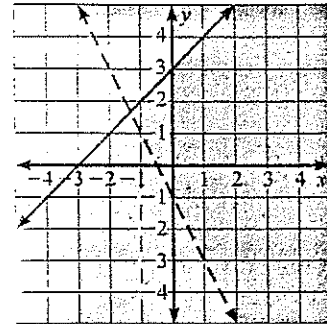
Recall that inequalities with  $<$  or  $>$  symbols are graphed with dashed lines, while inequalities with  $\leq$  or  $\geq$  symbols are graphed with solid lines.

**Graph the system of inequalities.**

$y \leq x + 3$	Inequality 1
$y > -2x - 1$	Inequality 2

**Solution:**

Graph both inequalities in the same coordinate plane. The graph of the system is the intersection of the two half-planes, which is shown as the darkest gray.



**PRACTICE**

**Graph the system of inequalities.**

- |                    |                   |                  |
|--------------------|-------------------|------------------|
| 1. $y > 4x + 1$    | 2. $y \geq x - 4$ | 3. $x - 2y < -4$ |
| $y < -x - 2$       | $y \leq -3x + 4$  | $y \geq -2$      |
| 4. $y \leq 2x - 2$ | 5. $x > -3$       | 6. $y > 1$       |
| $2x + 3y < 1$      | $x \geq -1$       | $x \leq 4$       |

## 2. Graphing Systems of Three Linear Inequalities

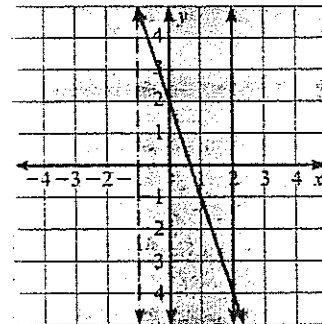
**EXAMPLE**

**Graph the system of inequalities.**

$y \leq -3x + 2$	Inequality 1
$x > -1$	Inequality 2
$x \leq 2$	Inequality 3

**Solution:**

Graph all three inequalities in the same coordinate plane. The graph of the system is the shaded region shown.



**PRACTICE**

**Graph the system of inequalities.**

- |                     |                       |                |
|---------------------|-----------------------|----------------|
| 7. $y \geq -2x - 1$ | 3. $y \geq x$         | 9. $y < x + 4$ |
| $y \geq 2x - 1$     | $x < 4$               | $4x + y < 4$   |
| $y > 4$             | $y > -1$              | $y \geq -4$    |
| 10. $y < 2$         | 11. $2x + 3y \leq -2$ | 12. $x < -3$   |
| $y > 2x$            | $y \geq -2x - 8$      | $x < 0$        |
| $y \leq -x + 3$     | $y > 3x + 6$          | $y \geq 0$     |

Choose a point in the shaded region and substitute it in each inequality. The solution checks if each substitution results in a true statement.

### 3. Write a System of Linear Inequalities

**EXAMPLE** Write a system of inequalities for the shaded region.

**Solution:**

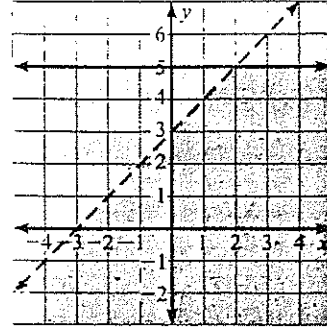
Find the equation of the lines from the slope and y-intercept, from two points, or from a point and the slope.

**Inequality 1** One boundary line for the shaded region is  $y = 5$ . Because the shaded region is *below* the *solid* line, the inequality is  $y \leq 5$ .

**Inequality 2** Another boundary line for the shaded region has a slope of 1 and a y-intercept of 3. So, its equation is  $y = x + 3$ . Because the shaded region is *below* the *dashed* line, the inequality is  $y < x + 3$ .

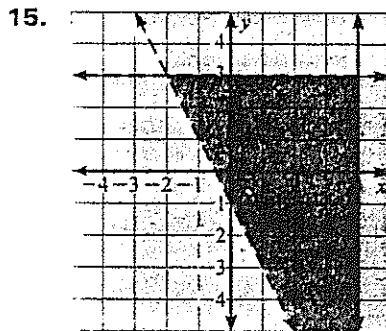
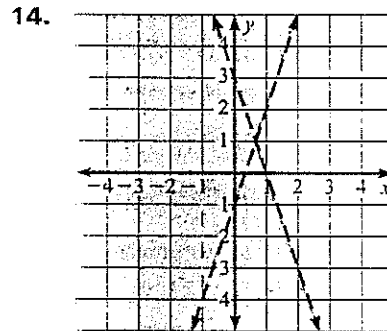
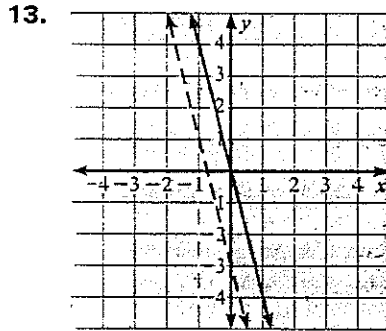
The system of inequalities for the shaded region is:

$$\begin{array}{ll} y \leq 5 & \text{Inequality 1} \\ y < x + 3 & \text{Inequality 2} \end{array}$$



**PRACTICE**

Write a system of inequalities for the shaded region.



### Quiz

Graph the system of inequalities.

1.  $x \leq -5$   
 $y \geq -1$

2.  $x + 2y > -1$   
 $y > -3$

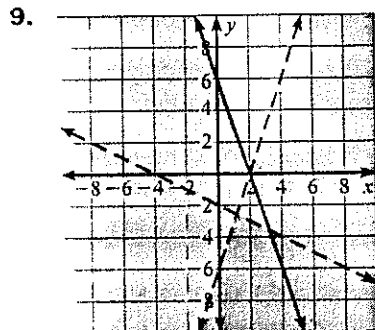
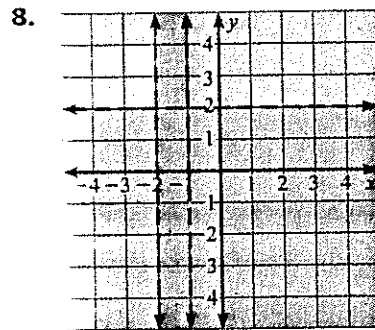
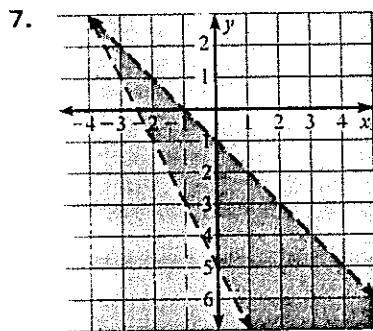
3.  $x + y \geq -2$   
 $x + y > -6$   
 $4x + y < -4$

4.  $y \geq -2$   
 $y \leq 1$   
 $x \geq 0$

5.  $x > 1$   
 $x > 2$   
 $x > 3$

6.  $y < -5x + 3$   
 $y \leq -4x - 2$   
 $4x - 3y < -3$

Write a system of inequalities for the shaded region.



# Solving Systems of Linear Inequalities

(pp. 67–69)

One way to solve a system of inequalities is to graph each inequality. The graph of the system is the intersection of these graphs.

## 1. Graphing Systems of Two Linear Inequalities

**Vocabulary**

**System of linear inequalities** Two or more linear inequalities in the same variables; also called a *system of inequalities*.

**EXAMPLE**

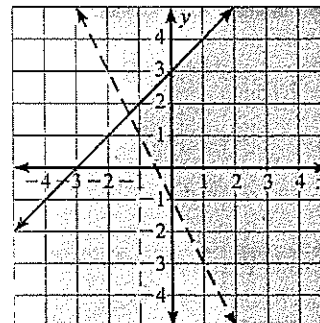
Recall that inequalities with < or > symbols are graphed with dashed lines, while inequalities with ≤ or ≥ symbols are graphed with solid lines.

**Graph the system of inequalities.**

$y \leq x + 3$	Inequality 1
$y > -2x - 1$	Inequality 2

**Solution:**

Graph both inequalities in the same coordinate plane. The graph of the system is the intersection of the two half-planes, which is shown as the darkest gray.



**PRACTICE**

**Graph the system of inequalities.**

- |                                     |                                       |                                 |
|-------------------------------------|---------------------------------------|---------------------------------|
| 1. $y > 4x + 1$<br>$y < -x - 2$     | 2. $y \geq x - 4$<br>$y \leq -3x + 4$ | 3. $x - 2y < -4$<br>$y \geq -2$ |
| 4. $y \leq 2x - 2$<br>$2x + 3y < 1$ | 5. $x > -3$<br>$x \geq -1$            | 6. $y > 1$<br>$x \leq 4$        |

## 2. Graphing Systems of Three Linear Inequalities

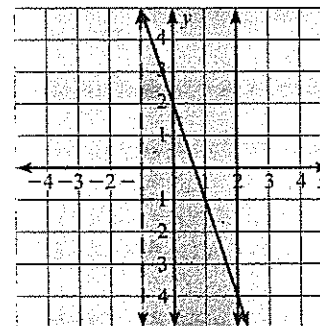
**EXAMPLE**

**Graph the system of inequalities.**

$y \leq -3x + 2$	Inequality 1
$x > -1$	Inequality 2
$x \leq 2$	Inequality 3

**Solution:**

Graph all three inequalities in the same coordinate plane. The graph of the system is the shaded region shown.



**PRACTICE**

**Graph the system of inequalities.**

- |                                                   |                                                           |                                               |
|---------------------------------------------------|-----------------------------------------------------------|-----------------------------------------------|
| 7. $y \geq -2x - 1$<br>$y \geq 2x - 1$<br>$y > 4$ | 8. $y \geq x$<br>$x < 4$<br>$y > -1$                      | 9. $y < x + 4$<br>$4x + y < 4$<br>$y \geq -4$ |
| 10. $y < 2$<br>$y > 2x$<br>$y \leq -x + 3$        | 11. $2x + 3y \leq -2$<br>$y \geq -2x - 8$<br>$y > 3x + 6$ | 12. $x < -3$<br>$x < 0$<br>$y \geq 0$         |

Choose a point in the shaded region and substitute it in each inequality. The solution checks if each substitution results in a true statement.



### 3. Write a System of Linear Inequalities

**EXAMPLE** Write a system of inequalities for the shaded region.

**Solution:**

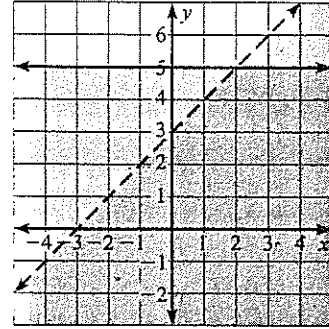
Find the equation of the lines from the slope and y-intercept, from two points, or from a point and the slope.

**Inequality 1** One boundary line for the shaded region is  $y = 5$ . Because the shaded region is *below* the *solid* line, the inequality is  $y \leq 5$ .

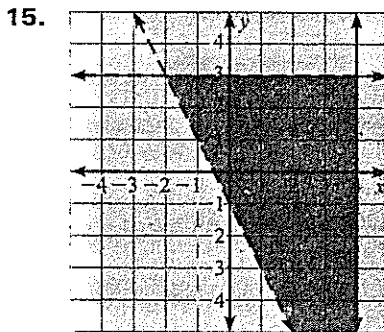
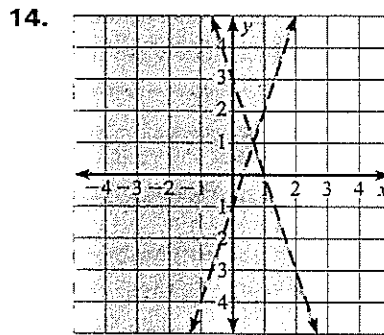
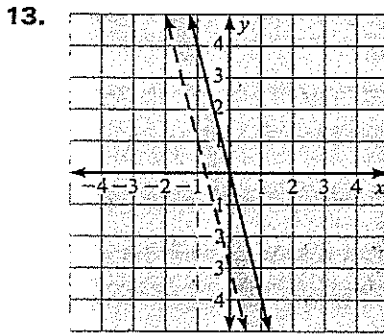
**Inequality 2** Another boundary line for the shaded region has a slope of 1 and a y-intercept of 3. So, its equation is  $y = x + 3$ . Because the shaded region is *below* the *dashed* line, the inequality is  $y < x + 3$ .

The system of inequalities for the shaded region is:

$$\begin{array}{ll} y \leq 5 & \text{Inequality 1} \\ y < x + 3 & \text{Inequality 2} \end{array}$$



**PRACTICE** Write a system of inequalities for the shaded region.



### Quiz

Graph the system of inequalities.

1.  $x \leq -5$   
 $y \geq -1$

2.  $x + 2y > -1$   
 $y > -3$

3.  $x + y \geq -2$   
 $x + y > -6$   
 $4x + y < -4$

4.  $y \geq -2$   
 $y \leq 1$   
 $x \geq 0$

5.  $x > 1$   
 $x > 2$   
 $x > 3$

6.  $y < -5x + 3$   
 $y \leq -4x - 2$   
 $4x - 3y < -3$

Write a system of inequalities for the shaded region.

